To eliminate y, we add the equations.

$$\begin{aligned} -10x + 2y &= -14 \\ -3x - 2y &= -12 \\ \hline -13x &= -26 \end{aligned}$$

Now, we can see that $x = 2$. This result can be used in either of the original equations to solve for y. Let's pick the first equation.

$$\begin{aligned} -10(2) + 2y &= -14 \\ -20 + 2y &= -14 \\ 2y &= 6 \\ y &= 3 \end{aligned}$$

And finally, we get the same solution as we got using substitution: $x = 2, y = 3$.

When solving systems of equations, you can use either method, but one of them will typically be faster. If you see a variable with no coefficient, like in $-5x + y = -7$ above, substitution is likely the best route. If you see matching coefficients, or you see that it's easy to get matching coefficients, elimination is likely the best route. The example above was simple enough for both methods to work well (though substitution was slightly faster). In these cases, it comes down to your personal preference.

Infinitely many solutions

In the *Other Types of Equation Questions* chapter, we learned that a single equation has infinitely many solutions when both sides of the equation are equivalent.

Similarly, a system of equations has infinitely many solutions when both equations are equivalent. Here, we have to clarify what we mean by "equivalent." Both equations in the system

$$\begin{aligned} 3x + 2y &= 5 \\ 3x + 2y &= 5 \end{aligned}$$

have exactly the same coefficients and constants, and therefore the system has infinitely many solutions ($(1, 1), (3, -2), (5, -5)$, to name just a few). However, the equations in the system

$$\begin{aligned} 6x + 4y &= 10 \\ 3x + 2y &= 5 \end{aligned}$$

do not have the same coefficients and constants, yet the system still has infinitely many solutions. Why is that? Because the two equations are *essentially* the same: the first equation can be divided by 2 to obtain the second equation. In other words, the two equations are equivalent.

Two equations are equivalent when one equation can be manipulated to form the other. A system has infinitely many solutions when the equations in the system are equivalent.

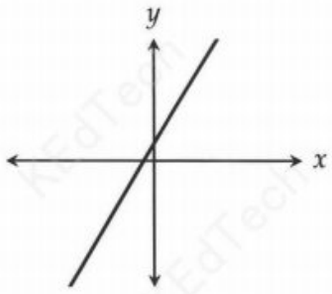
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What about the graphs of systems that have infinitely many solutions or no solution?

Graphing the following system,

$$2y - 4x = 2$$
$$y - 2x = 1$$

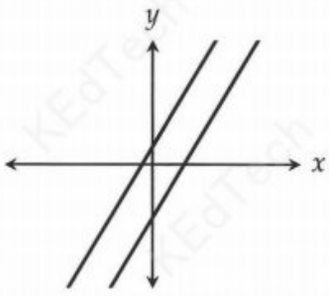
which has infinitely many solutions because its equations are equivalent, we get



It's just one line! Well, actually it's two lines, but because they're the same line, they overlap and intersect at an infinite number of points. Hence, an infinite number of solutions.

Now let's graph a system with no solution:

$$y - 2x = 1$$
$$y - 2x = -3$$



What do you notice about the lines? They're parallel! They have no intersection points. Hence, no solution.

EXAMPLE 7: In the xy-plane, the graphs of the equations $y = 3x - 5$ and $y = -2x + 10$ intersect at the point (h, k). What is the value of k ?

As mentioned earlier, the solutions to a system of equations are the intersection points of the graphs of those equations, and vice versa. **So to find the point(s) where two graphs intersect, solve the system consisting of their equations.** In this problem, that system is

$$y = 3x - 5$$
$$y = -2x + 10$$

Substituting the first equation into the second, we get

$$3x - 5 = -2x + 10$$
$$5x = 15$$
$$x = 3$$

When $x = 3$, $y = 3(3) - 5 = 4$. So the two lines intersect at $(3, 4)$ and $k = \boxed{4}$.

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EXAMPLE 8:

$$y = x^2 - 5x + 6$$
$$y = x + 1$$

The graphs of the equations in the given system of equations intersect at the point (x, y) in the xy-plane. What is a possible value of y ?

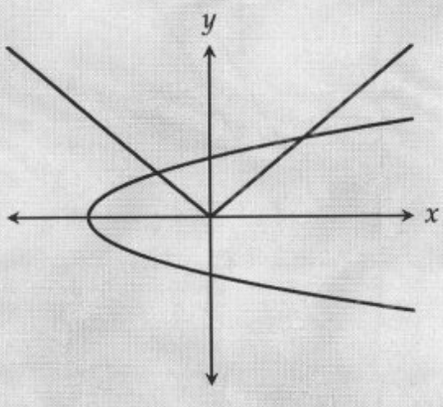
The solutions to the system are the intersection points, so let's solve the system. Substituting the first equation into the second, we get

$$x^2 - 5x + 6 = x + 1$$
$$x^2 - 6x + 5 = 0$$
$$(x - 1)(x - 5) = 0$$
$$x = 1 \text{ or } 5$$

When $x = 1, y = 1 + 1 = 2$. When $x = 5, y = 5 + 1 = 6$. So the graphs of the two equations intersect at $(1, 2)$ and $(5, 6)$, which means the possible values of y are $\boxed{2}$ and $\boxed{6}$.

EXAMPLE 9:

$$y^2 = x + 3$$
$$y = |x|$$



The given system of equations is represented in the graph. How many solutions does the system have?

A) Zero B) Exactly one C) Exactly two D) Infinitely many

Simple. The graphs intersect at two points, so the system has two solutions. Answer $\boxed{(C)}$.

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EXERCISE 1: Answers for this exercise start on page 360.

1

$$y = -5x + 65$$
$$y = 25$$

Which ordered pair (x, y) is a solution to the given system of equations?

A) $(-8, 25)$
B) $(8, 25)$
C) $(25, -8)$
D) $(25, 8)$

2

$$3x - 5y = -11$$
$$x = 1 - 3y$$

The solution to the given system of equations is (x, y). What is the value of y ?

A) -1
B) 0
C) 1
D) 2

3

$$y + 2x = 20$$
$$6x - 5y = 12$$

What is the solution (x, y) to the given system of equations?

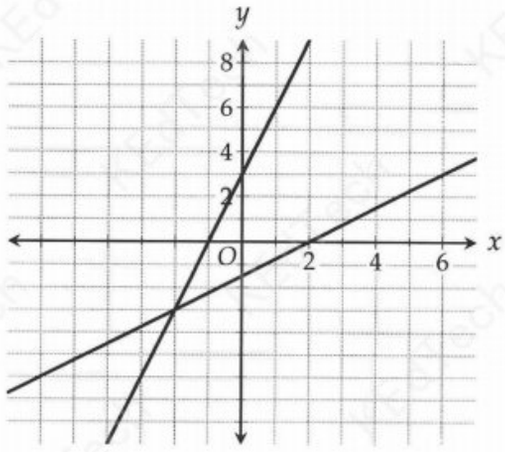
A) $(-7, 6)$
B) $(-6, 7)$
C) $(6, 7)$
D) $(7, 6)$

4

$$y = x^2 + 10x + 23$$
$$x - 9 = -15$$

The solution to the given system of equations is (x, y). What is the value of y ?

5



The graph of a system of two linear equations is shown. What is the solution (x, y) to the system?

A) $(-4, -6)$
B) $(-3, -2)$
C) $(-2, -3)$
D) $(-1, 2)$

6

$$2x + 5y = 24$$
$$x + 4y = 15$$

The solution to the given system of equations is (x, y). What is the value of $x + y$?

A) 7
B) 8
C) 9
D) 10

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7

$$2x - 4y = 8$$
$$x + 2y = 4$$

How many solutions (x, y) does the given system of equations have?

A) Zero
B) One
C) Two
D) More than two

8

An ice cream vendor sells small and large cups of ice cream. Each small cup of ice cream is sold for \$5, and each large cup is sold for \$7. During an event, the vendor sold a total of 200 cups of ice cream and collected \$160 more from sales of small cups than from sales of large cups. If x is the number of small cups sold at the event and y is the number of large cups sold at the event, which system of equations represents this situation?

A) $x - y = 160$
$5x + 7y = 200$

B) $x - y = 160$
$7x + 5y = 200$

C) $x + y = 200$
$5x = 7y + 160$

D) $x + y = 200$
$7y = 5x + 160$

9

If $\frac{a}{b} = 4.5$ and $\frac{a}{kb} = 12.5$, what is the value of k ?

10

$$6x - 3y = 9$$

One of the two linear equations in a system is given. The system has no solution. Which equation could be the second equation in the system?

A) $3x - y = 2$
B) $2x - y = 3$
C) $\frac{1}{3}x - \frac{1}{6}y = 2$
D) $\frac{1}{3}x - \frac{1}{6}y = \frac{1}{2}$

11

$$x^2 - y^2 = 48$$
$$x - y = 6$$

If (x, y) is the solution to the given system of equations, what is the value of $x + y$?

A) 8
B) 12
C) 16
D) 24

12

$$2x - 5y = a$$
$$bx + 10y = -8$$

In the given system of equations, a and b are constants. If the system has infinitely many solutions, what is the value of a ?

A) -4
B) $\frac{1}{4}$
C) 4
D) 16

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13

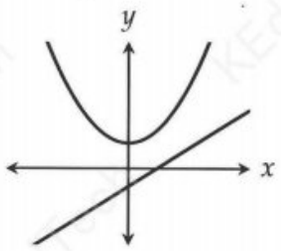
$$y = ax + b$$
$$y = -bx$$

In the given system of equations, a and b are constants. The graphs of the equations in the xy-plane intersect at the point $(2, 8)$. What is the value of a ?

A) 2
B) 4
C) 6
D) 8

14

$$y = x^2 + 1$$
$$y = x - 1$$



The given system of equations is represented in the graph. How many solutions does the system have?

A) Zero
B) Exactly one
C) Exactly two
D) Exactly three

15

$$3x - \frac{5}{16} = 4$$
$$y = 2\left(3x - \frac{5}{16}\right)^2$$

The solution to the given system of equations is the ordered pair (x, y). What is the value of y ?

A) 8
B) 16
C) 32
D) 64

16

$$ax + 2y = 5$$
$$3x - 6y = 20$$

In the given system of equations, a is a constant. The system has exactly one solution. Which of the following could NOT be the value of a ?

A) -1
B) $\frac{3}{4}$
C) 1
D) 3

17

$$3x = 16$$
$$80 = 4y^2 + 3x$$

The graphs of the equations in the given system of equations intersect at the point (x, y) in the xy-plane. What is a possible value of y ?

A) $-\frac{16}{3}$
B) -4
C) $\frac{4}{3}$
D) 8

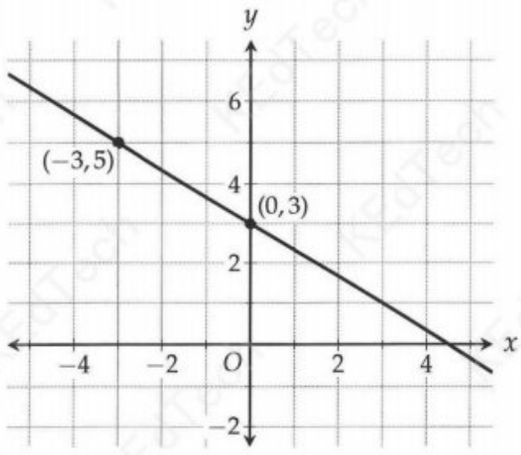
18

$$-5x = 8 - ky$$
$$18y + 5x = -10x + 21$$

In the given system of equations, k is a constant. If the system has no solution, what is the value of k ?

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19



The graph of one of the two equations in a linear system is shown in the xy-plane. The system has no solution. Which equation could be the second equation in this system?

A) $y = -\frac{3}{2}x - 1$

B) $y = \frac{3}{2}x + 3$

C) $y = -\frac{2}{3}x - 1$

D) $y = -\frac{2}{3}x + 3$

20

In the xy-plane, the graph of $y = x^2 - 7x + 7$ intersects the graph of $y = 2x - 1$ at the points $(1, 1)$ and (p, q). What is the value of p ?

21

Shannon drove 340 miles from Melrose to Norwich, passing Hanover along the way. From Melrose to Hanover, her average speed was 50 miles per hour, and from Hanover to Norwich, her average speed was 30 miles per hour. The drive took 9 hours. How many miles did Shannon drive from Melrose to Hanover?

M → N : 340 50 m/h

H → N 30 m/h

M → H → N $50x + 30(9-x) = 340$

x 9-x $50x + 270 - 30x = 340$

$20x = 70$ $x = 3.5$

22

$$\frac{4}{3} + \frac{5}{4}x = 28y + \frac{5}{8}x$$

$$my = \frac{1}{2}(5x - 8)$$

In the given system of equations, m is a constant. If the system has no solution, what is the value of m ?

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EXERCISE 2: Answers for this exercise start on page 363.

1

$$4x - \frac{1}{3}y = -8$$
$$y = 4x + 16$$

Which ordered pair (x, y) is the solution to the given system of equations?

A) $(-2, 8)$
B) $(-1, 12)$
C) $(1, 20)$
D) $(3, 28)$

2

$$9x = -7y + 22$$
$$5x = 7y - 76$$

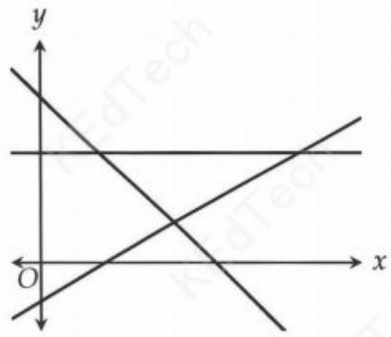
The solution to the given system of equations is (x, y). What is the value of $7x$?

3

$$y = 0.5x + 14$$
$$x - y = -18$$

The solution to the given system of equations is (x, y). What is the value of y ?

4



A system of three linear equations is represented in the graph. How many solutions does the system have?

A) Zero
B) Exactly one
C) Exactly two
D) Exactly three

5

Which of the following systems of linear equations has no solution?

A) $y = 8$
$x = 8$

B) $y = 8$
$y = 8x + 8$

C) $y = 8x$
$y = 8x + 8$

D) $y = -8x + 8$
$y = 8x + 8$

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6

$$y = 2x - 14$$
$$y = (x + 5)(x - 7)$$

The solution to the given system of equations is (x, y). What is a possible value of x ?

A) -7
B) -3
C) 4
D) 9

7

$$3x - 6y = 15$$
$$-2x + 4y = -10$$

At how many points do the graphs of the given equations intersect in the xy-plane?

A) Zero
B) One
C) Two
D) More than two

8

$$y = -10x$$
$$x^2 + y = 96$$

A solution to the given system of equations is (x, y), where $y < 0$. What is the value of y ?

9

If $\frac{x+y}{x} = 9$ and $\frac{ay}{3x} = 32$, where a is a constant, what is the value of a ?

10

$$9(2x + 1) + 27(y - 4) = 810$$
$$(2x + 1) + (y - 4) = -393$$

The solution to the given system of equations is (x, y). What is the value of $8(y - 4)$?

11

A restaurant has two types of tables, rectangular ones that can each seat 4 people and circular tables that can each seat 8 people. If 144 people are enough to fill all 30 tables at the restaurant, how many rectangular tables does the restaurant have?

A) 12
B) 16
C) 20
D) 24

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12

$$mx - 6y = 10$$
$$2x - ny = 5$$

In the given system of equations, m and n are constants. The system has infinitely many solutions. What is the value of $\frac{m}{n}$?

A) $\frac{1}{12}$

B) $\frac{1}{3}$

C) $\frac{4}{3}$

D) 3

13

One of the equations in a system of two linear equations is $x - 2y = 5$. The system has exactly one solution. Which equation could be the other equation in the system?

A) $-x + 2y = 0$

B) $x - 2y = 0$

C) $-x + 2y = -5$

D) $x + 2y = -5$

14

$$x^2 - y^2 = \frac{1}{12}$$
$$x - 2y = 0$$

If (x, y) satisfies the given system of equations, what is a possible value of y ?

A) $-\frac{1}{6}$

B) $-\frac{1}{\sqrt{12}}$

C) $\frac{1}{4}$

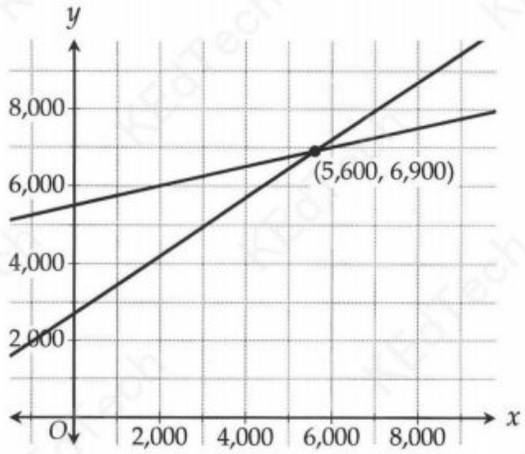
D) $\frac{1}{2}$

15

$$\frac{3}{10}y + \frac{b}{3}x = \frac{3}{4} - \frac{1}{5}y$$
$$\frac{3}{8}x + \frac{3}{5} = -\frac{1}{4}y + \frac{8}{5}$$

In the given system of equations, b is a constant. If the system has no solution, what is the value of b ?

16



What system of linear equations is represented by the lines shown?

A) $-\frac{x}{0.01} + \frac{y}{0.04} = 90$
$-40x + 30y = 220$

B) $-\frac{x}{0.01} + \frac{y}{0.04} = 220$
$-40x + 30y = 90$

C) $-0.01x + 0.04y = 90$
$-\frac{x}{40} + \frac{y}{30} = 220$

D) $-0.01x + 0.04y = 220$
$-\frac{x}{40} + \frac{y}{30} = 90$

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17

The function f is defined by $f(x) = ax^3 + bx$, where a and b are constants. In the xy-plane, the graph of $y = f(x)$ passes through the points $(-1, -10)$ and $(2, 38)$. What is the value of ab ?

A) 9
B) 21
C) 24
D) 39

18

$$x^2 + 2x = y - 1$$
$$-4x = y + 8$$

The solution to the given system of equations is (x, y). What is the value of x ?

19

When Kenji uses his push mower, he mows 57.75 square yards of grass per minute, and when he uses his self-propelled mower, he mows 80.25 square yards of grass per minute. If Kenji spent a total of 2 hours using his two mowers in a week and mowed a total of 9,000 square yards of grass, how many minutes did he spend using his push mower?

x $2-x$

$57.75x + 80.25(2-x) = 9000$

28 minutes

20

$$y = \sqrt{x} + 3$$
$$\sqrt{4x} - y = 3$$

The solution to the given system of equations is (x, y). What is the value of y ?

21

$$-15x + 3y = 12$$
$$y + 132 = x^2 - 4x$$

The graphs of the given equations in the xy-plane intersect at the point (x, y). What is a possible value of x ?

22

$$5x - 3y = 8$$
$$-20x + 12y = -32$$

Which of the following ordered pairs (x, y) is a solution to the given system of equations for all real values of n ?

A) $\left(n, \frac{5}{3}n - 8\right)$
B) $\left(-\frac{3}{5}n + \frac{8}{5}, n\right)$
C) $\left(n - 1, \frac{5}{3}n - 1\right)$
D) $\left(\frac{3}{5}n + 1, n - 1\right)$

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